

AFCRL -65-572

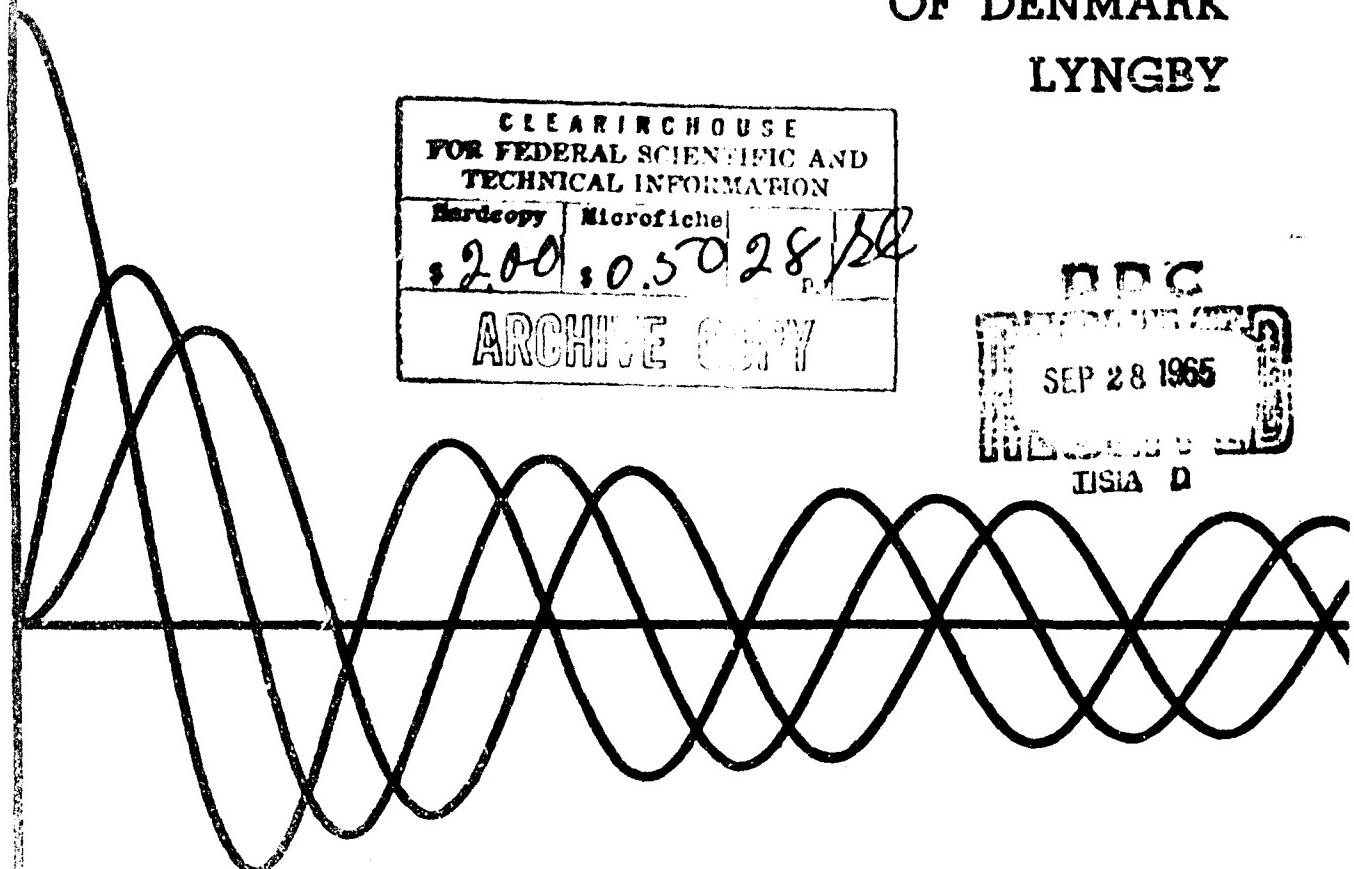
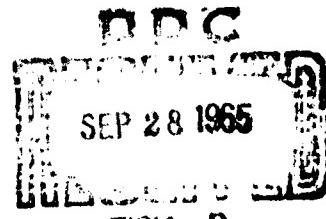
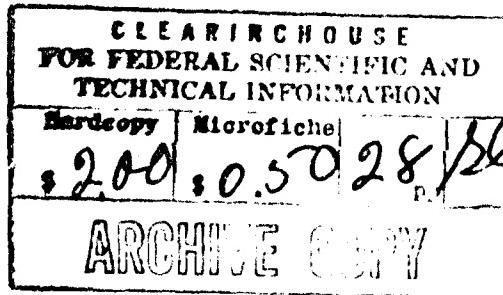
Best Available Copy

Laboratory  
of  
ELECTROMAGNETIC THEORY

AD621048



THE TECHNICAL UNIVERSITY  
OF DENMARK  
LYNGBY



Reflector Arrays

Tove Larsen

Annual Summary Report                    S 127 R 45  
Contract AF 61(052)-794                April 1965

The research reported in this document has been sponsored  
by the AIR FORCE CAMBRIDGE RESEARCH LABORATORIES, OAR under  
Contract AF 61(052)-794 through the European Office of Aerospace  
Research, (OAR), United States Air Force.

A scheme for a theoretical investigation of an arbitrary Van Atta reflector with or without a conducting plate is given. A four element linear reflector consisting of half wave dipoles has been given special attention. The reradiation pattern of such reflector has been examined analytically and numerically and the theoretical results have been confirmed by experiment. It is found that a Van Atta reflector do not always behave as stated in the patent description.

TABLE OF CONTENTS

Abstract .....	1
1. Introduction .....	3
2. General considerations .....	4
3. Arbitrary Van Atta reflector in free space .....	5
4. Linear four element Van Atta reflector .....	7
4.1 Theoretical results .....	7
4.2 Experimental results .....	8
5. Arbitrary four element reflector .....	9
6. Two-dimensional reflector with a plate .....	10
7. Unidirectionally conducting disk .....	12
8. Conclusion .....	13
9. Literature .....	14

Errata for reports SR 1 and SR 2

Figures

The purpose of the present contract has been to investigate the properties of a Van Atta reflector, the most simple form of an adaptive antenna system. The investigations have been carried out along two lines.

First a scheme has been set up for a theoretical and numerical treatment of an arbitrary Van Atta reflector. The method is described in section 3 for a reflector in free space and in section 6 for a reflector with a conducting plate placed parallel to the antenna elements. In section 7 the theory has been applied to the special problem of reflection from a unidirectionally conducting disk.

The other way of approach was to consider, theoretically, numerically and experimentally, a rather simple, special form of a Van Atta reflector. A linear, four-element dipole reflector was chosen for this purpose. The analysis led to some general results given in section 2, which are valid for all Van Atta reflectors. The basic analysis is outlined in section 4, and in section 5 it is investigated what could be obtained with a four-element, not necessarily linear, four element dipole reflector with unequally spaced elements and unequal lengths and characteristic impedances of the transmission lines.

## 2. GENERAL CONSIDERATIONS

A Van Atta reflector is a passive reflector consisting of a number of antenna elements mutually connected with transmission lines of equal, but arbitrary length. It was patented by L.C. Van Atta<sup>1)</sup> in 1959, and since then a number of papers with suggestions for improvements and utilizations of the reflector has been given. However, the basic form of the reflector has only been given little attention. A survey of the literature on Van Atta reflectors is given in SR 1<sup>2)</sup> of this contract.

The basic idea of the reflector is that the energy absorbed by one element is transmitted to its connected mate and reradiated from this. This causes a reflected field propagating back in the direction of arrival of the incoming wave for all angles of incidence. The principle is sketched in fig.

3. In this explanation two facts are neglected, the mutual impedances between the elements and the field reradiated by the elements due to the current induced in themselves by the primary wave. As shown in SR 2<sup>3)</sup> this field is of the same order of magnitude as the field reradiated by the mates and will cause a reflected wave propagating in the mirror direction as related to the incoming wave, see fig. 4. So two reflected waves are found from a Van Atta reflector, and as the total reradiation pattern depends on the relative phases of these waves both the distance between adjacent elements and the length of the transmission lines will influence the pattern.

### 3. ARBITRARY VAN ATTA REFLECTOR IN FREE SPACE

The method of investigation of an arbitrary Van Atta reflector is described in SR 1<sup>2)</sup>.

The antenna elements are dipoles placed on an (imaginary) smooth surface, fig. 5. An appropriate coordinate system is introduced to characterize the position and orientation of the antennas and the characteristics of the primary plane wave.

The open circuit voltage induced at the terminals of an element is given by

$$v_n = \bar{E} \cdot \bar{L}_{\text{eff}}, \quad (1)$$

where  $\bar{E}$  is the electric field strength at the position of the element, and  $\bar{L}_{\text{eff}}$  the effective length<sup>4)</sup> of the elements.

Each pair of elements is equivalent with a circuit as shown in fig. 6. The antennas are represented by a generator with the voltage found above in series with the selfimpedance  $Z_A$  of the antenna and a number of equivalent generators due to the mutual impedances. This representation is valid if the scattered field of the open-circuited antenna is small compared to that of the terminated antenna<sup>5)</sup>; this is the case for half wave dipoles and smaller dipoles. The transmission lines are represented by X-circuits which are applicable for all values of the length of the lines.

Using ordinary circuit theory the following set of equations is obtained for a pair of antennas of an array with N elements:

$$(v_m - v_k) \cos \frac{ka}{2} = (-i \sin \frac{ka}{2} + (z_A - z_{mk}) \cos \frac{ka}{2})(i_m - i_k) \\ + \sum_{\substack{n=1 \\ n \neq m, k}}^N i_n (z_{nm} - z_{nk}) \cos \frac{ka}{2}, \quad (2)$$

$$(v_m + v_k) \sin \frac{ka}{2} = (i \cos \frac{ka}{2} + (z_A + z_{mk}) \sin \frac{ka}{2})(i_m + i_k) \\ + \sum_{\substack{n=1 \\ n \neq m, k}}^N i_n (z_{nm} + z_{nk}) \sin \frac{ka}{2}, \quad (3)$$

where normalized values of impedances, voltages and currents have been introduced and where  $a$  is the length of the transmission line and  $k$  the propagation constant.

A Van Atta reflector with  $N$  elements will lead to a system of  $N$  equations with complex coefficients in the  $N$  unknown currents. This matrix equation may be solved by an electronic computer, and from the currents found the reradiation pattern is calculated using the theory of antenna arrays.

#### 4. LINEAR FOUR ELEMENT REFLECTOR

The theoretical and numerical investigation of this reflector is given in SR 2<sup>3)</sup> and the experimental investigation in SR 3<sup>6)</sup>.

##### 4.1. Theoretical investigation

The reflector investigated is shown in fig. 3. The method of analysis is similar to the one given in SR 1, but because of the limited number of elements it has been possible, in the case where mutual impedances are neglected, to obtain an expression for the reradiated field, which was examined analytically. In the case where mutual impedances are taken into account the analysis was performed numerically.

When mutual impedances are neglected the following results were found:

1. The reradiation pattern is symmetrical about the normal to the reflector. This means, that if the reflector has a maximum of reradiation back in the direction of arrival it also works as a mirror.
2. The maximum reflection is not always back in the direction of arrival.
3. The reradiation pattern depends on the length of the transmission lines, in some cases there will be no reflection at all.

These results do not agree with the statements given in the patent description.

Further the following special patterns have been considered for a reflector with the distance  $0.5\lambda$  between elements,  $\lambda$  being the wavelength:

1. Grazing and normal incidence.
2. Cases where only the phase of the reflected field depends on the length of the transmission lines.
3. The length of the transmission lines being an integral number of a half or a full wavelength.
4. The reradiation patterns are the same for  $a = \lambda/2 + \alpha\lambda + p\lambda$  and  $a = \lambda/2 - \alpha\lambda + p\lambda$ , where  $p$  is an integer and  $\alpha$  an arbitrary number.
5. Optimum transmission line length.

When mutual impedances are taken into account the results found above will be changed to some degree. The reradiation patterns will be symmetrical about the normal to the reflector only when the length of the transmission lines is a multiplum of half wavelengths. The coupling between the antennas may either support the Van Atta effect or the mirror effect depending on the length of the transmission lines and the angle of incidence.

As an example of the results mentioned is in fig. 7 shown the reradiation pattern for a reflector with the transmission lines being an integral number of

wavelengths. It is seen that there is no reradiation at all for normal incidence. Further is in fig. 8 shown how the coupling influences the radiation patterns as described above. Finally is in fig. 9 and 10 shown how the results found deviate from the idea of the patent. ( $\alpha_i$  and  $\alpha_u$  are angles of incidence and reradiation, respectively, see fig. 3).

#### 4.2. Experimental investigation

The measurements were performed at  $\lambda = 9,35$  cm in a radioanechoic box.

The four half-wave dipoles of the Van Atta reflector are slot fed dipoles with open-end terminations. Line-stretchers are inserted in the transmission lines connecting the dipoles in order to investigate the influence of the length of the lines. A photo of the reflector with line-stretchers is shown in fig. 1.

The experimental arrangement is shown in fig. 11 and a photo of the reflector in the anechoic chamber is shown in fig. 2. The reflector is placed on a moveable pedestal in the middle of the box. The measurements are based on the principle of interference between two signals, the signal to be measured and an inevitable signal reflected from the walls of the box. The last mentioned signal is controlled by a direct connection between transmitter and receiver.

For angles of incidence being  $30^\circ$ ,  $60^\circ$ ,  $70^\circ$  and  $90^\circ$  with respect to the plane of the reflector reradiation patterns are measured for angles up to  $\pm 70^\circ$  with the direction of incidence, while the length of the connecting transmission lines is changed at least a wavelength.

From these measurements the following properties of the reflector could be found:

1. Maximum reradiation is not always back in the direction of arrival of the incident wave.
2. The reflector has a mirror effect to the same degree as it has a Van Atta effect.
3. The reradiation depends on the length of the transmission lines.
4. The radiation patterns are assymetrical.

These results are all in agreement with the theoretical results found above. As an example is in fig. 12 shown some experimental results compared with the corresponding theoretical ones.

## 5. ARBITRARY FOUR ELEMENT REFLECTOR

Still using four parallel half-wave dipoles in the same plane, but letting the position of the elements and the length of the transmission lines variate, a computer program has been made in order to find an optimum form of the reflector. The criterion for optimum has been to let the resulting antenna currents as much as possible be in phase for reradiation back in the direction of arrival. The investigation is not finished yet, but the results obtained now indicate , that the transmission lines should be of the same length.

If the procedure leads to a suitable optimum a reflector with the optimum characteristics found will be built and investigated experimentally.

## 6. TWO - DIMENSIONAL REFLECTOR WITH A PLATE

A two-dimensional Van Atta reflector investigated experimentally by Sharp, Fusca and Diab<sup>7) 8) 9)</sup> consisted of 16 half-wave dipoles mounted one quarter of a wavelength above a conducting plate. In order to compare the theoretical results with these experiments, and as a system like this has the advantage of avoiding unknown reflections from set-ups behind the reflector, it is considered worth while to investigate the influence of such plate.

The reflecting system then consists of two devices, the dipoles and the plate. The reflecting properties of the plate are supposed not to be influenced by the presence of the dipoles. The dipole system will be treated along the same lines as in section 3, but the induced voltage, the mutual impedances and the determination of the reradiated field have to be changed because of the presence of the plate. In order to be able to perform these calculation we have to assume that the plate is infinite.

The procedure outlined above is similar to the use of image theory except that the latter results in a reflected plane wave instead of the field reflected from the plate alone.

The induced voltage is still found from equation (1), but  $\bar{E}$  has to be changed because of the plate. The new value of  $\bar{E}$  is found from ordinary reflection theory.

The new values of self- and mutual impedances are found using the method of images. From fig. 13a we have

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad (4)$$

As  $I_2 = -I_1$  we obtain

$$Z_{11 \text{ plate}} = \frac{V_1}{I_1} = Z_{11} - Z_{12} \quad (5)$$

From fig. 13b we have the relation

$$V_2 = Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3 + Z_{24}I_4 \quad (6)$$

Putting  $I_2 = I_4 = 0$ ,  $I_3 = -I_1$  we find

$$Z_{12 \text{ plate}} = \frac{V_2}{I_1} = Z_{21} - Z_{23} \quad (7)$$

Using the values of induced voltage and of self- and mutual impedances found above in the system of equations (23) of SR 1, the currents of the antenna elements may be found. Next the reradiation pattern of the reflector without the plate but with the currents found, may be calculated. Eventually the final reradiation pattern is found using the theory of antenna arrays on the array consisting of two parallel Van Atta reflectors in free space with the distance  $2h$ , where  $h$  is the distance from the antennas to the plate.

In order to find the total field reflected from the Van Atta reflector we have to add to the field found above the field reflected from the plate itself. This field is found by the method given f.ex. in Kerr's book<sup>10)</sup>, where the back scattering cross section of a rectangular plate is given for a plate the dimensions of which are not small compared to the wavelength.

## 7. UNIDIRECTIONALLY CONDUCTING DISK

As a by-product of the investigations of Van Atta reflectors it has been tried to apply the theory found in section 3 to find the back scattering cross section of a unidirectionally conducting disk. Such disk has been investigated by Toraldo di Francia<sup>11)</sup> using diffraction theory. His result is a power series in  $kr$  where  $r$  is the radius of the disk, so this result is only applicable for small values of  $r$  compared to the wavelength.

The disk is considered as a Van Atta reflector with short-circuited elements. A computer program is being worked out to find the back scattering cross section of this reflector for various angles of incidence, radius of the disk and number of elements. In order to avoid infinite values of the self reactance of the elements it has to be assumed that they have a non-zero thickness.

The investigation is not yet finished. It is not expected that very exact results will be obtained, but it was considered worth while to try to treat a known problem by a somewhat unusual method, and it might give results which are useful for larger disks, where the more exact theory is not valid.

#### 8. CONCLUSION

A survey has been given of the work carried out until now on Van Atta reflectors by this laboratory. The work comprises a theory for treating an arbitrary Van Atta reflector with or without a conducting plate. Further a four-element linear reflector with half-wave dipoles has been investigated theoretically, numerically and experimentally. Good agreement is obtained between theory and experiments. It is found that a Van Atta reflector do not always behave as stated in the patent discription. Among other things it is found that maximum reradiation is not always back in the direction of the incident wave, and that the reflector shows a mirror effect as well as a Van Atta effect. The reradiation depends on the length of the transmission lines and will in some cases be zero.

## 9. LITERATURE

1. L.C. Van Atta, "Electromagnetic Reflector". U.S. Patent No. 290 800 2, October 6, 1959.
2. Tove Larsen, "A theoretical investigation of Van Atta arrays". Scientific Report No. 1, Contract No. AF 61(052)-794, Laboratory of Electromagnetic Theory, Technical University of Denmark, Nov. 1964.
3. J. Appel-Hansen, "Linear Van Atta reflector consisting of four half-wave dipoles". Scientific Report No. 2, Contract No. AF 61(052)-794, Laboratory of Electromagnetic Theory, Technical University of Denmark, Nov. 1964.
4. George Sinclair, "The transmission and reception of elliptically polarized waves". Proc. I.R.E. (1950) 148-151.
5. R.F. Harrington, "Electromagnetic scattering by antennas". IEEE Trans. Antennas and Propagation (1963) 595-596.
6. J. Appel-Hansen, "Experimental investigation of a linear Van Atta reflector". Scientific Report No. 3, Contract No. AF 61(052)-794, Laboratory of Electromagnetic Theory, Technical University of Denmark, May 1965.
7. E.D. Sharp, "Properties of the Van Atta reflector array". Rome Air Dev. Center technical report 58-53, AD 148684, April 1958.
8. J.A. Fusca, "Compact reflector has e.c.m. potential". Aviation Week, p. 66-69, January 5, 1959.
9. E.D. Sharp and M.A. Diab, "Van Atta reflector array". IRE Trans. PGAP, Vol. AP-8, p. 436-438, 1960.
10. D.E. Kerr, "Propagation of short radio waves". M.I.T. Rad. Lab. Ser. Vol. 13 p. 456. Mc. Graw Hill 1951.
11. G. Toraldo di Francia, "On a macroscopic measurement of the spin of electromagnetic radiation". Il Nuovo Cimento, Ser X. Vol. 6, p. 150-167, 1957.

Errata to SR 1 of Contract AF 61(052)-794

- p 4 1 11 f t      tvist → first  
p 5 1 13 f t      dimensioned → dimensional  
p 5 1 13 f t      on quarter → one quarter  
p 5 1 2 f b      refelction → reflection  
p 6 1 9 f t      duscussion → discussion  
p 7 1 9 f t      courses are → course is  
p 7 1 9 f b       $(x_n, y_n, z_n) \rightarrow (x_n, y_n, z_n)$   
p 7 1 11 f t      are → is  
p 8 1 1 f t      the exponent should read  
                       $-ik(x_n \cos\phi_i \sin\theta_i + y_n \sin\phi_i \sin\theta_i + z_n \cos\theta_i)$   
p 8 1 5 f t      quality → quantity  
p 8 1 6 f t      for dipole → for a dipole  
p 8 (4)            first term of numerator should read  
                       $\cos(\frac{\pi}{\lambda} L \cos\alpha)$   
p 8 1 4 f b      antenn → antenna  
p 9 1 2 f t      to the own → to its own  
p 9 1 11 f t      antennas → antenna  
p 9 1 12 f t      an parallel → and parallel  
p 9 1 10 f b      with sinusoidal current distribution  
p 11 1 6 f t       $z_n \rightarrow z_N$   
p 11 1 5 f b       $(2p + \lambda)\frac{1}{4} \rightarrow (2p + 1)\frac{\lambda}{4}$   
p 11 1 4 f b       $(2p + \lambda)\frac{1}{2} \rightarrow (2p + 1)\frac{\lambda}{2}$   
p 12 1 12 f b       $L_{S1}^+ \rightarrow z_{S1}^+$   
p 12 1 4 f b      brachets → brackets  
p 14 1 12 f t       $T(\theta, \phi) \rightarrow \sigma(\theta, \phi)$   
p 15 1 2 f t      preceeings → preceeding  
p 15 1 3 f t      induced → included  
p 15 1 5 f t       $(L = \lambda/2)$   
p 15 1 8 f t       $z_o \rightarrow Z_o$   
p 16 1 8 f b       $z_A \rightarrow Z_A \rightarrow z_o \rightarrow Z_o$   
p 17 1 3 f t      investigation → investigating  
p 17 1 6 f b      og → of  
p 19 1 4 f t      pt. → M.  
p 19 1 7 f t      Engineering → Engineer

p 19 1 5 f b Scatteres + Scatterer  
p 19 1 5 f b Enhancement + Enhancement  
p 20 1 6 f t porarized + polarized  
p 20 1 9 f b Stearning + Stearns  
p 20 1 6 f b Ollsen + Olesen  
**Fig. 6** (25) + (24)  
(26) + (25)  
(31) + (30)  
(27) + (26)  
(28) + (27)  
(29) + (28)  
(30) + (29)  
(32) + (31)  
(33) + (32)

Errata to SR 2 of Contract AF 61(052)-794

p 1 1 2 f t        Theoretical + theoretical  
p 3 1 12 and 3 f b analytic + analytical  
p 4 1 17 f t        field + some fields  
p 6 1 5 f t        are + is  
p 7 1 3 f t        consideration + considerations  
p 10 1 12 f t      refelctor + reflector  
p 14 1 9 f t        atempt + attempt  
p 14 1 1 f b        minor + mirror  
p 16 (1A)           R<sub>an</sub> + R<sub>An</sub>  
p 16 1 7 f t        R<sub>an</sub> + R<sub>An</sub>  
p 16 (1A), 1 7 and 8 f t Z<sub>an</sub> + Z<sub>An</sub>  
p 16 1 7 f t        Where + where  
                      imput + input

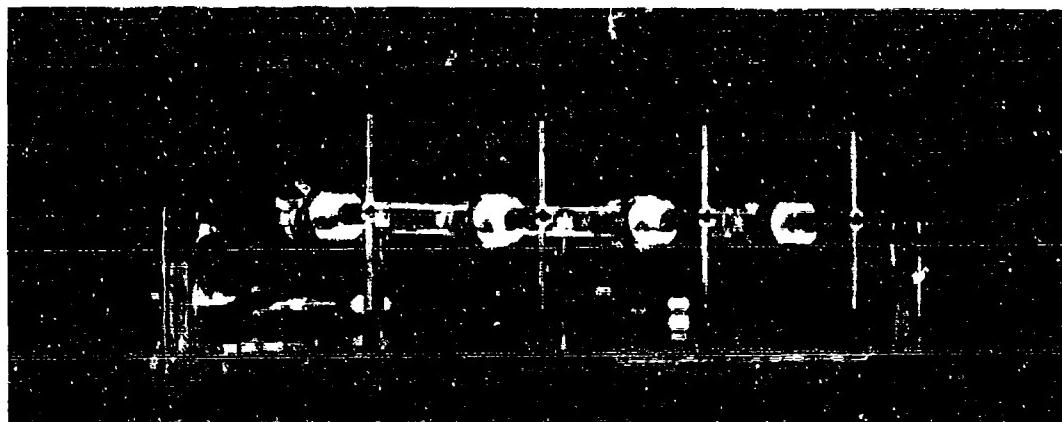


Fig. 1. Four element Van Atta reflector

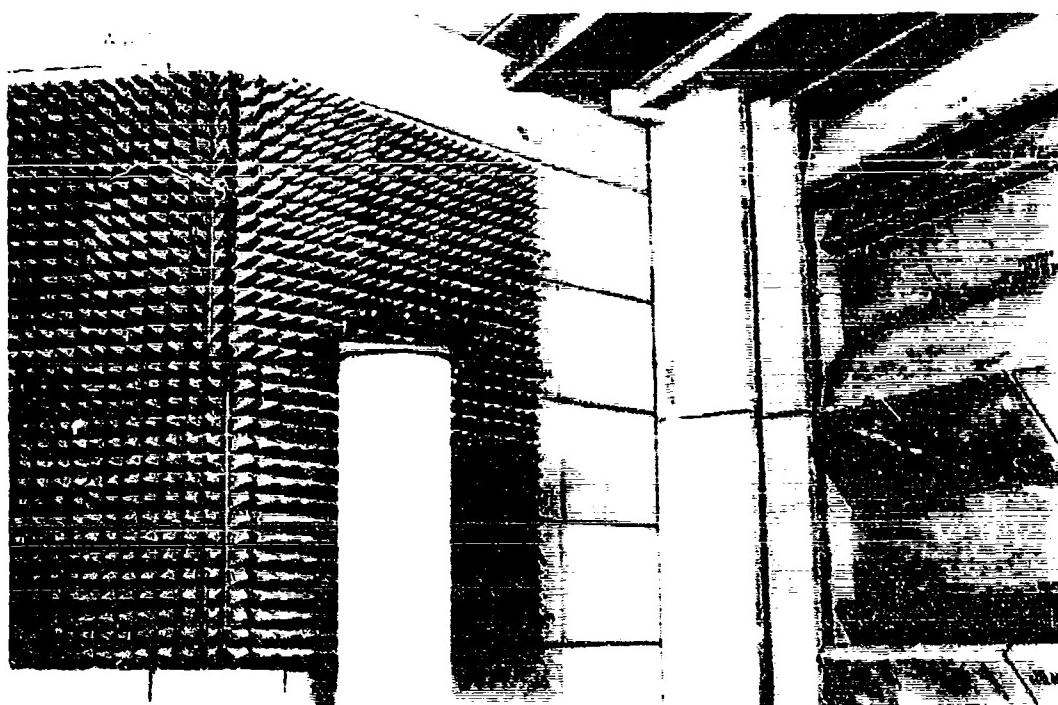
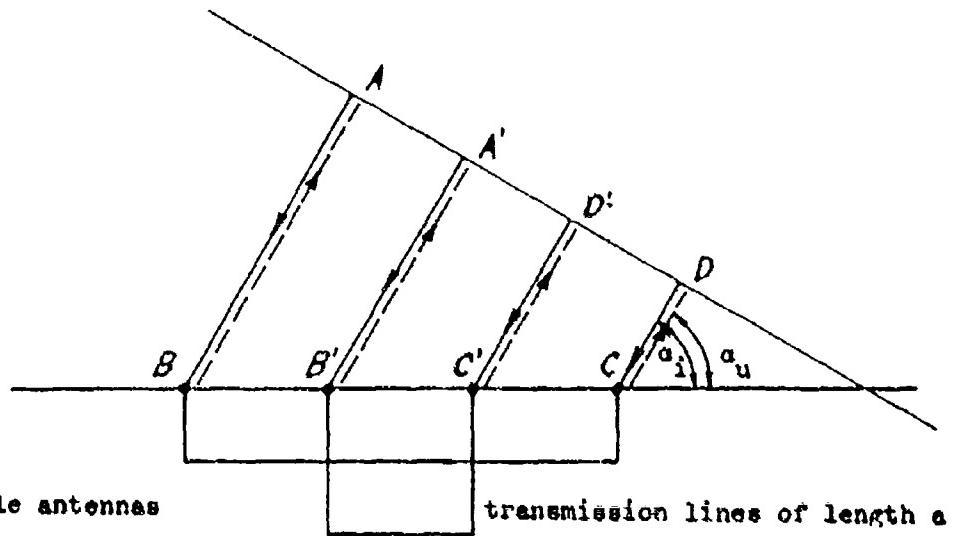
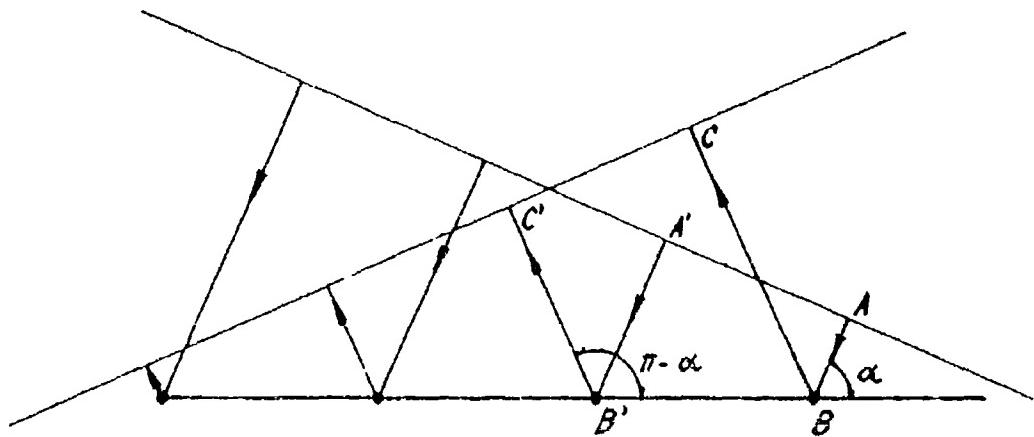


Fig. 2. Van Atta reflector placed on pedestal in  
radioanechoic box



*fig 3. The Van Atta effect.  
The paths ABCD and A'B'C'D' are equal.*



*fig 4. The mirror effect.  
The paths ABC and A'B'C' are equal.*

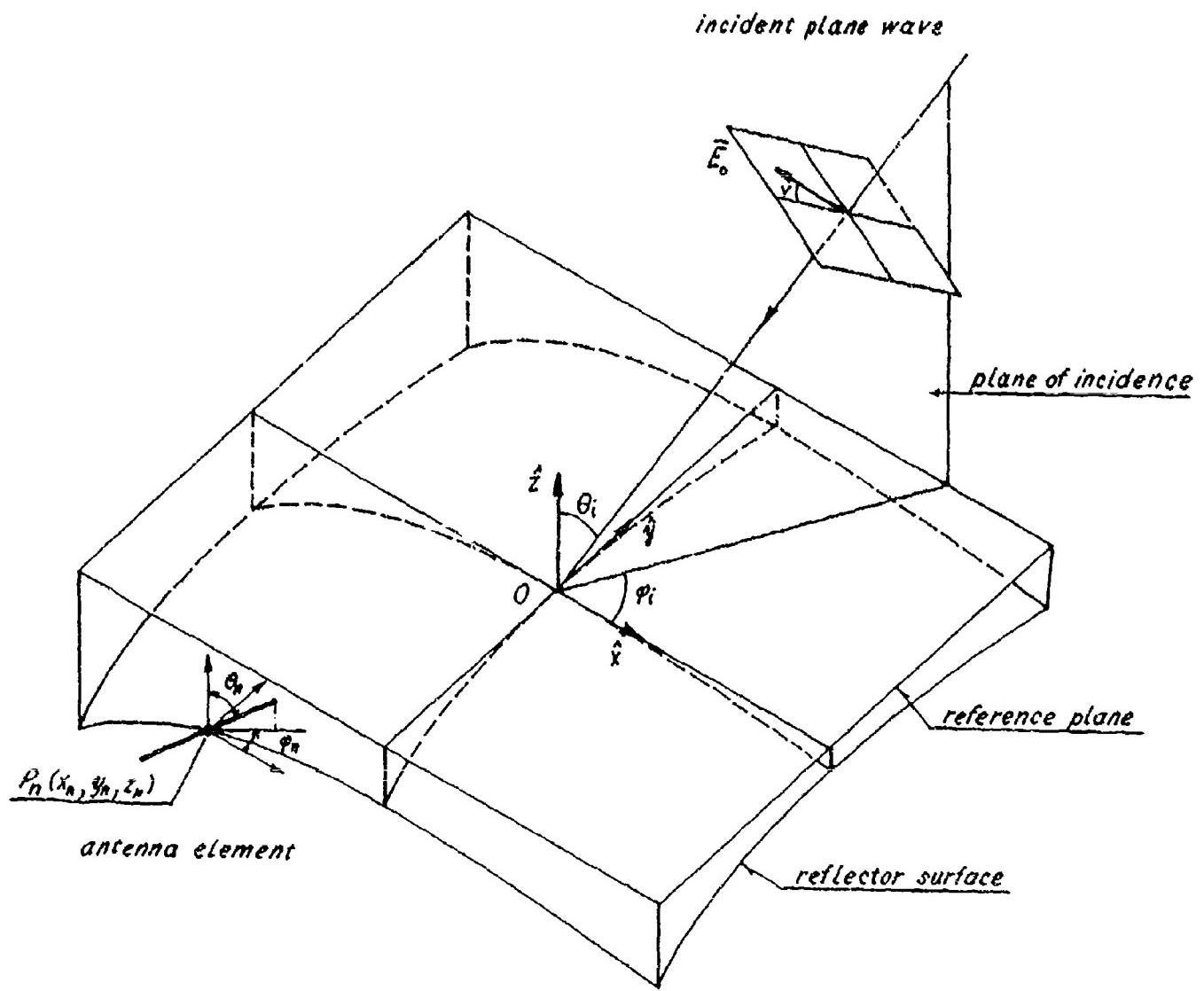


Fig 5. Coordinate system for reflector surface

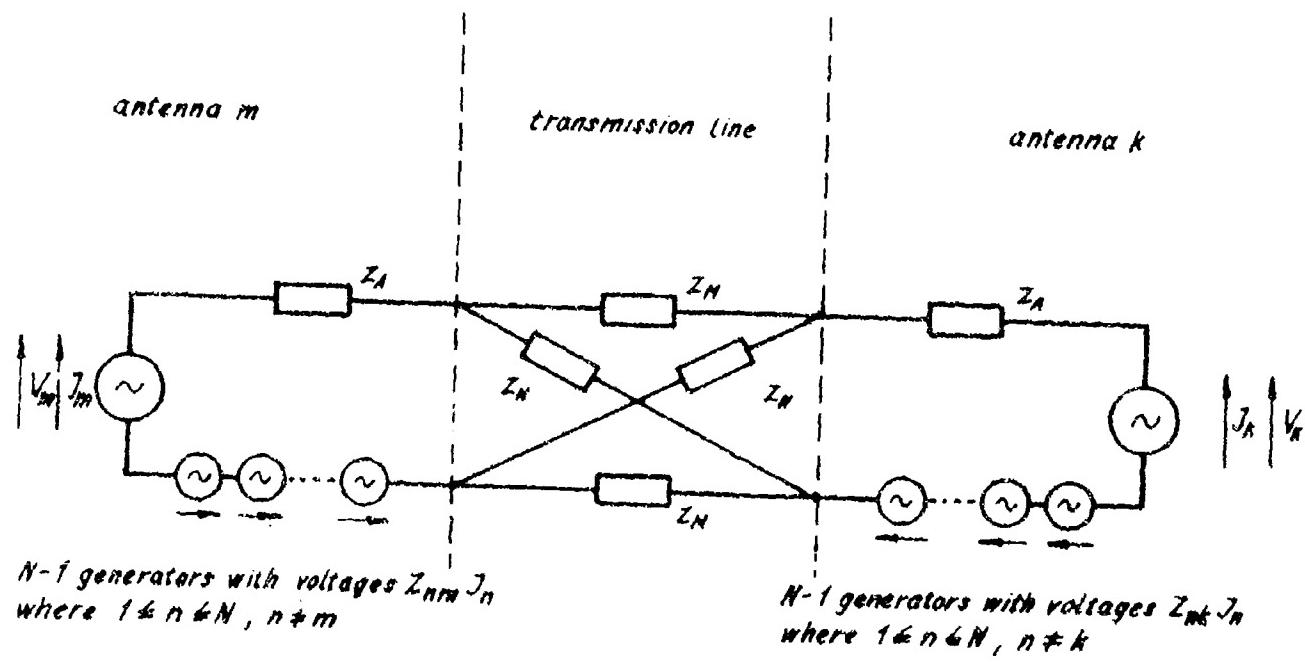


Fig 6 . Equivalent circuit of two interconnected array elements

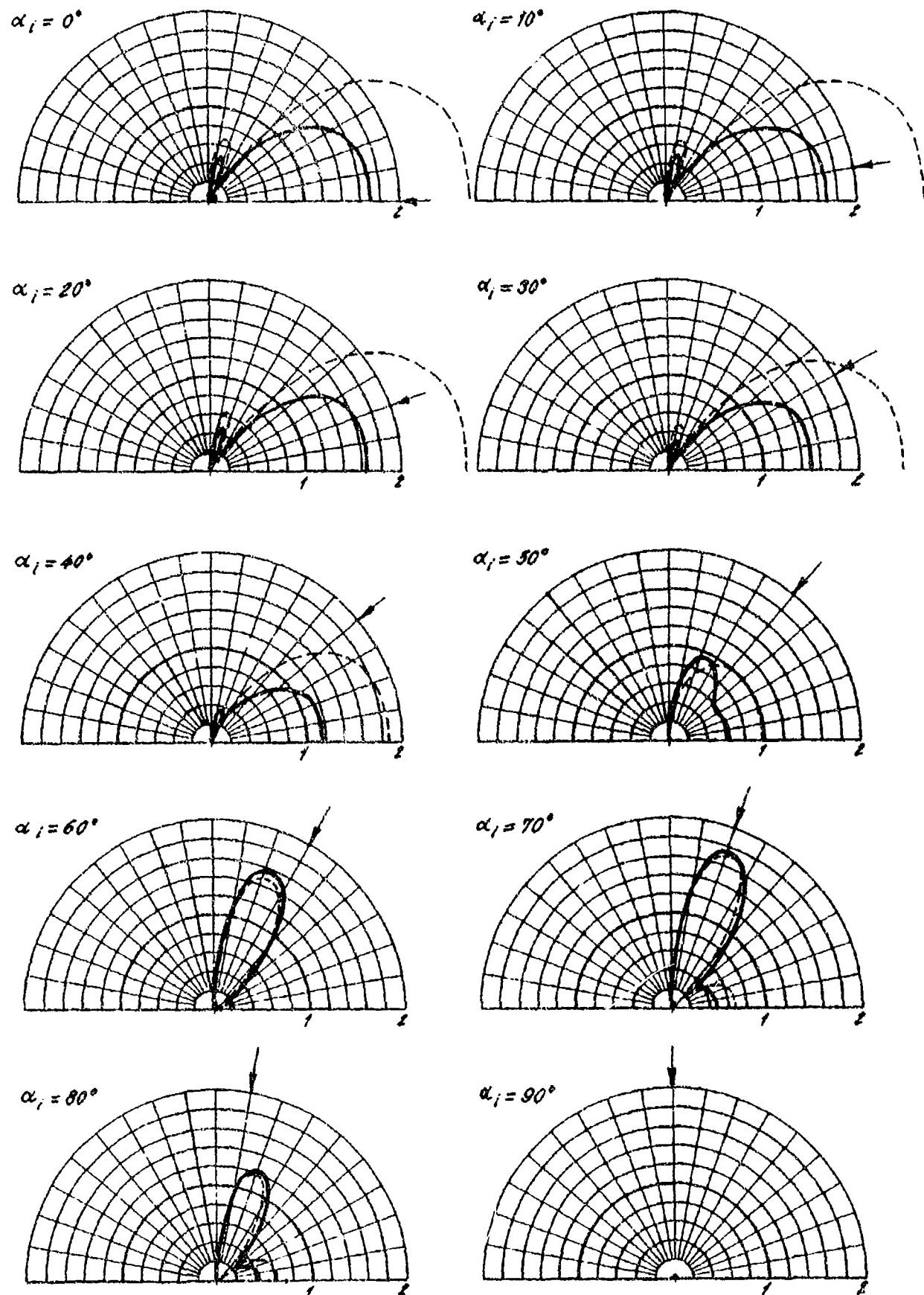


Fig 7. Radiation patterns when  $a = 1\lambda$ .

---- mutual impedances neglected.

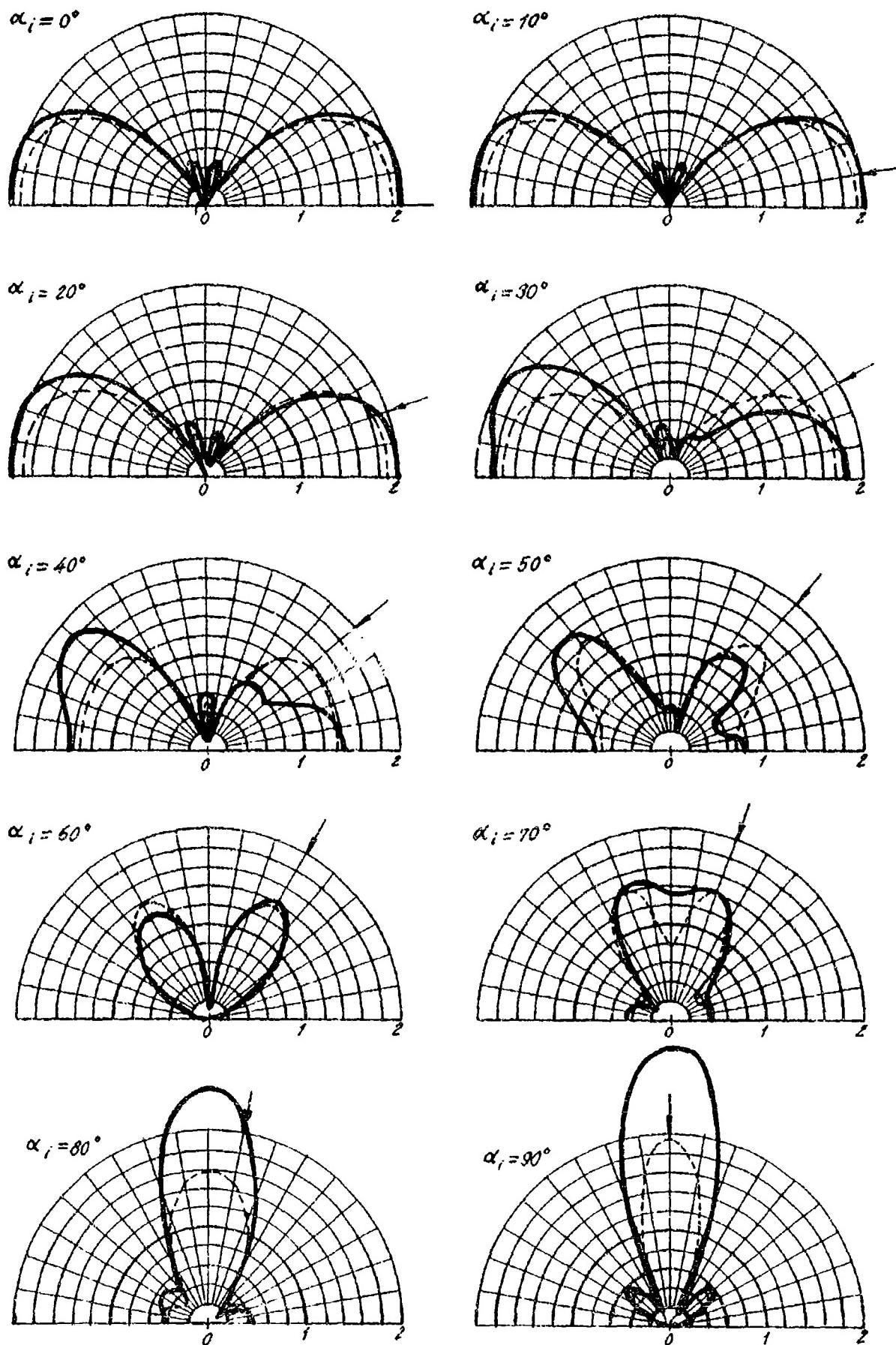


Fig 8. Radiation patterns when  $\alpha = 0,75\lambda$ .

----- mutual impedance normalized

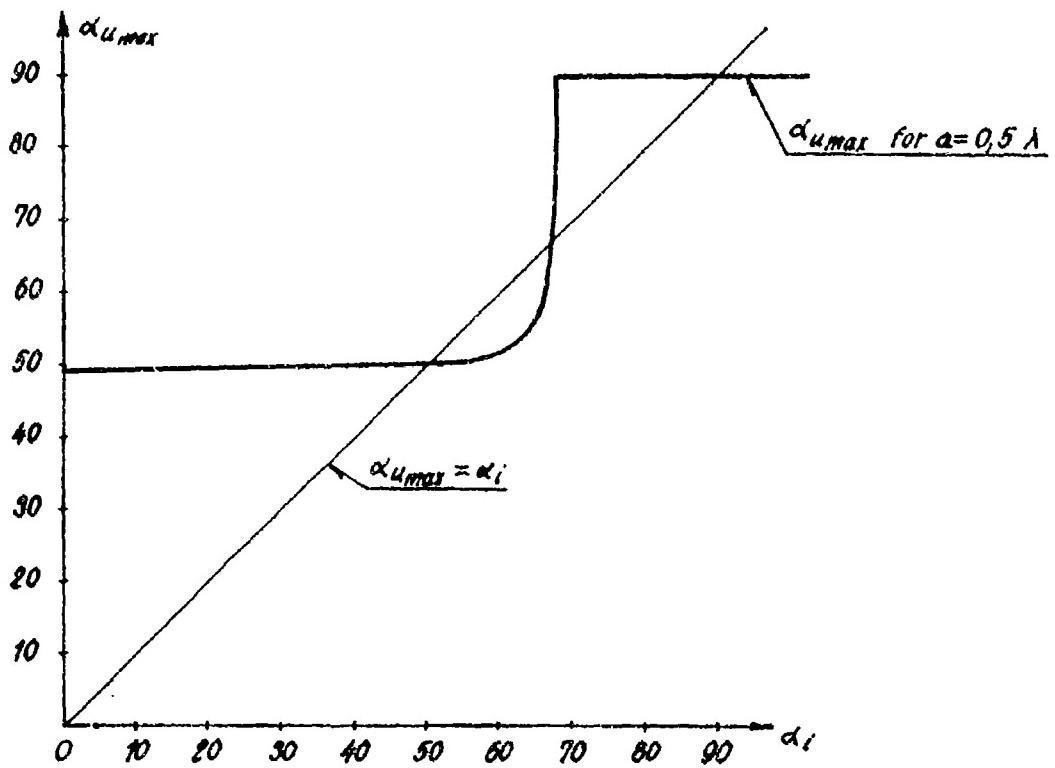


fig 9.  
 $\alpha_{u\max}$  versus  $\alpha_i$ , when  $a = 0.5 \lambda$

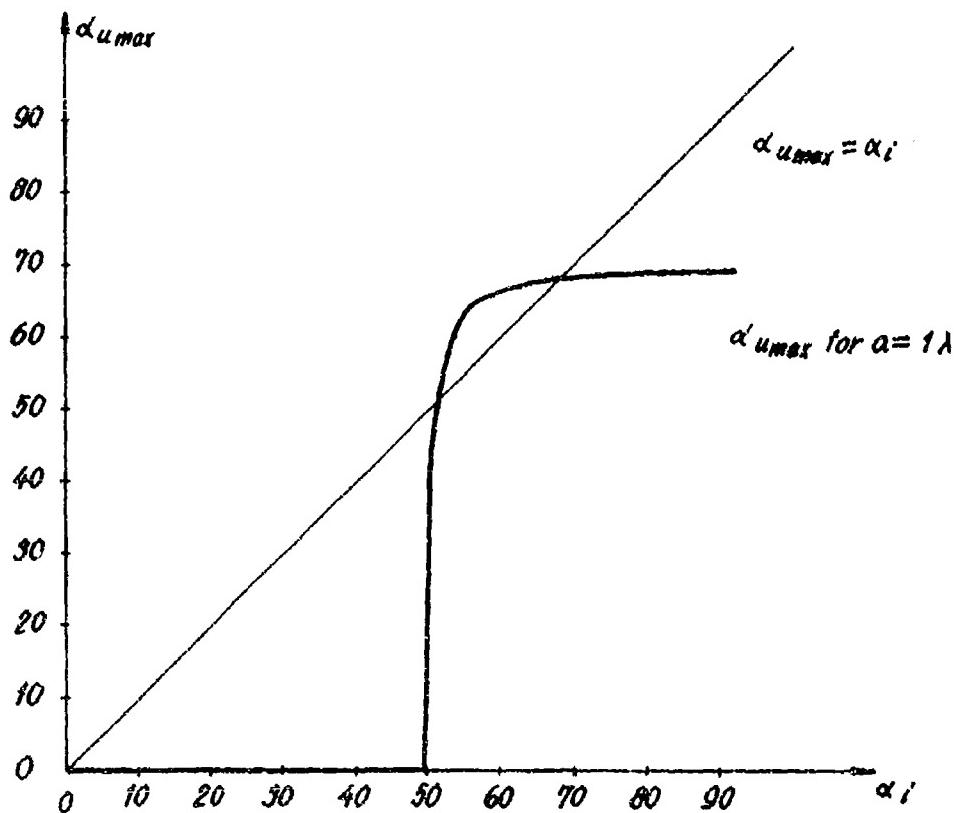


fig 10.

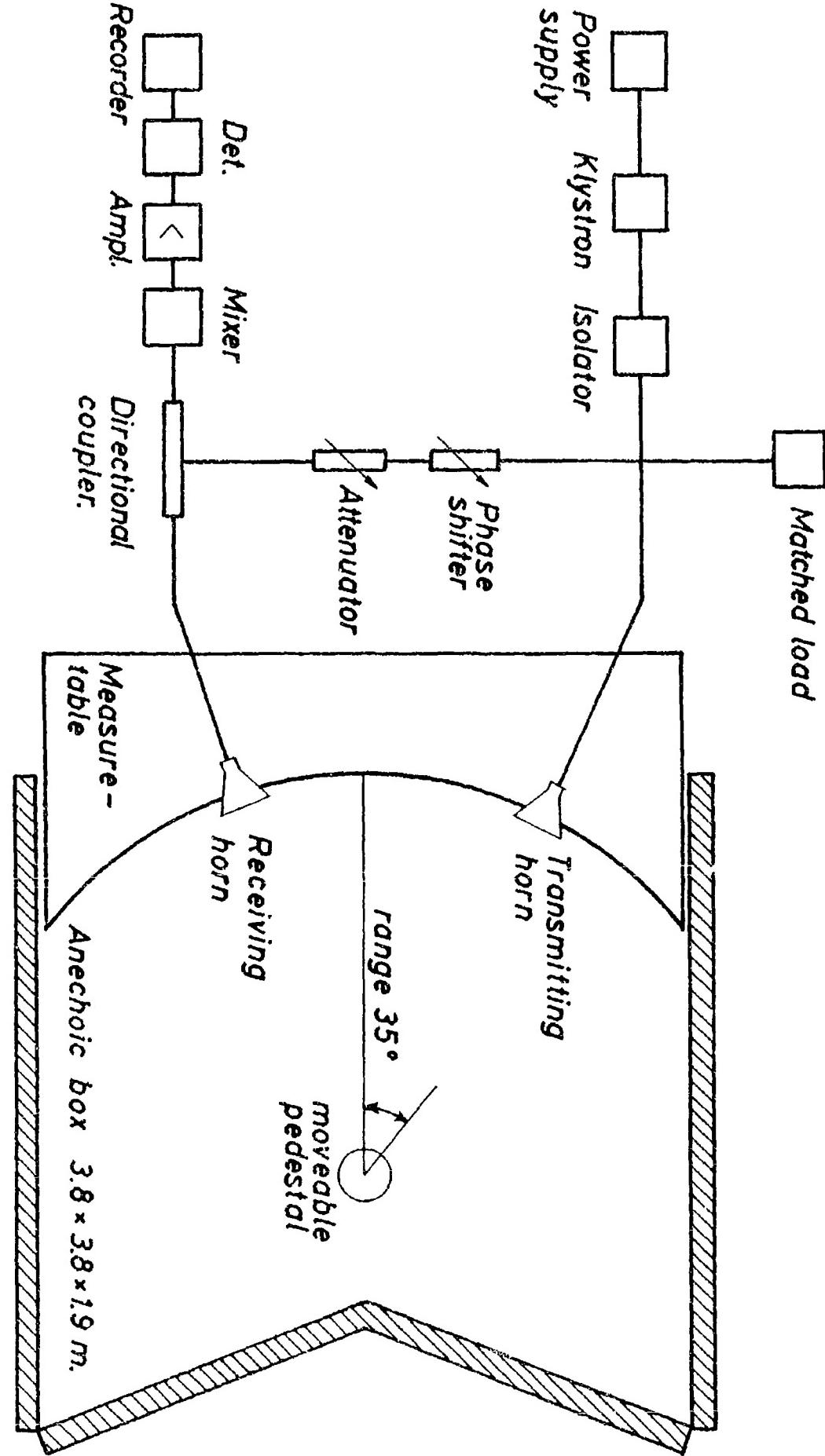
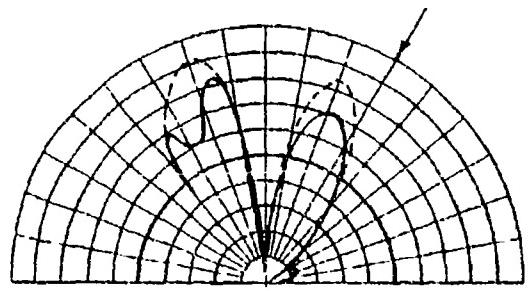
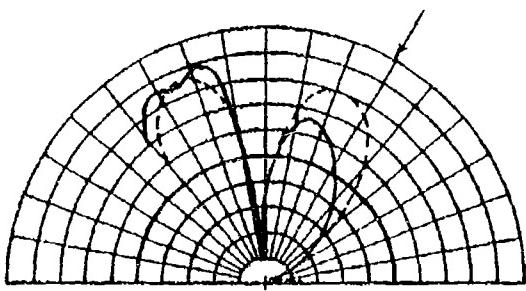


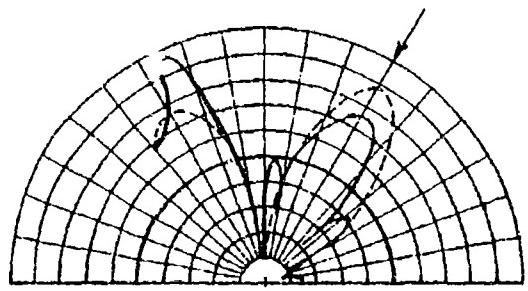
Fig. 11 Experimental arrangement



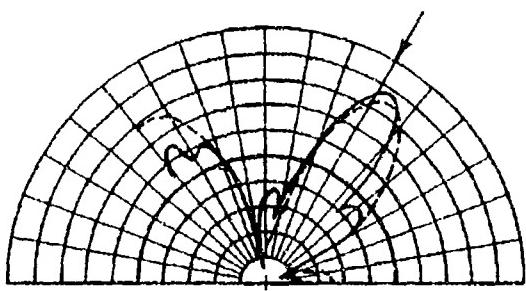
$$a = 0,08 \lambda + p \lambda$$



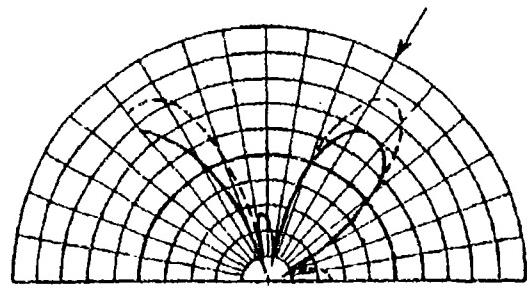
$$a = 0,19 \lambda + p \lambda$$



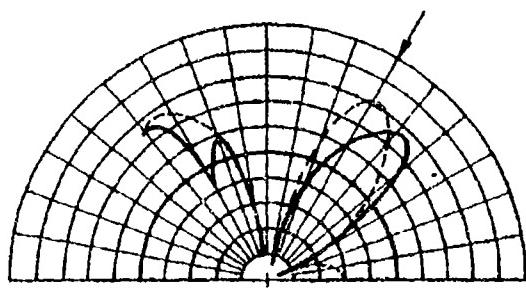
$$a = 0,30 \lambda + p \lambda$$



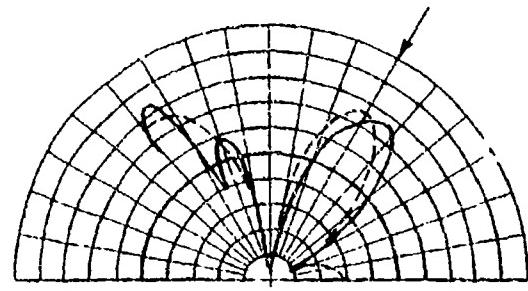
$$a = 0,41 \lambda + p \lambda$$



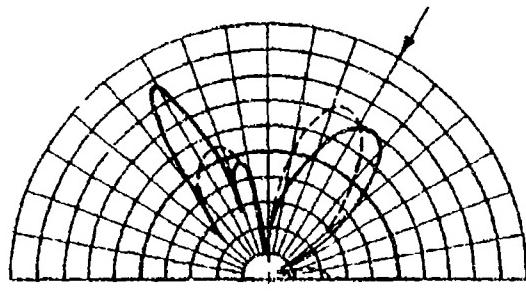
$$a = 0,52 \lambda + p \lambda$$



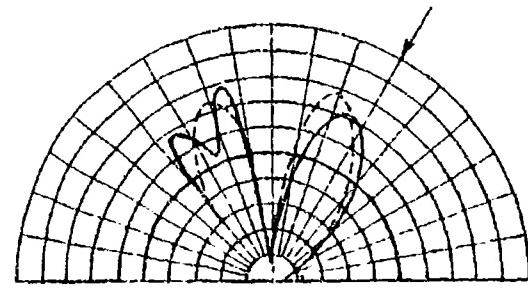
$$a = 0,63 \lambda + p \lambda$$



$$a = 0,74 \lambda + p \lambda$$



$$a = 0,85 \lambda + p \lambda$$



$$a = 0,96 \lambda + p \lambda$$

— experimental.

- - - theoretical.

Fig. 12. Radiation patterns of a parabolic antenna.

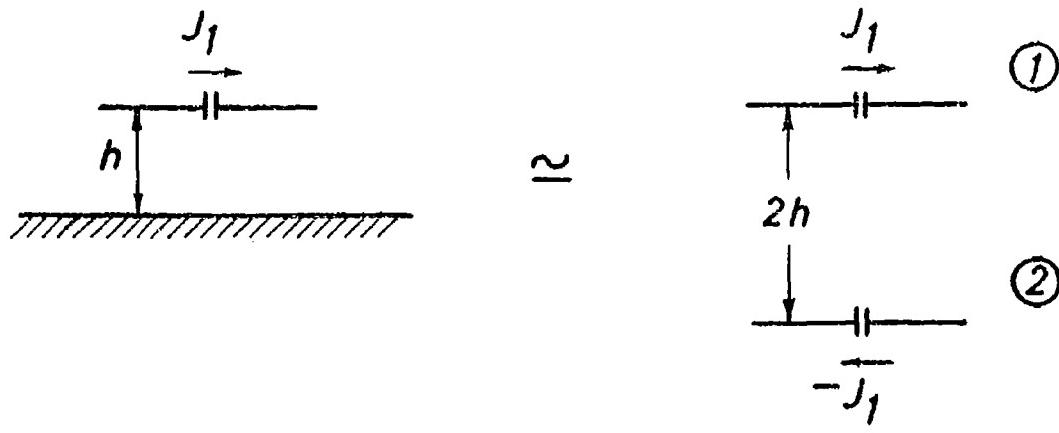


Fig. 13a. Self impedance of dipole above plate.

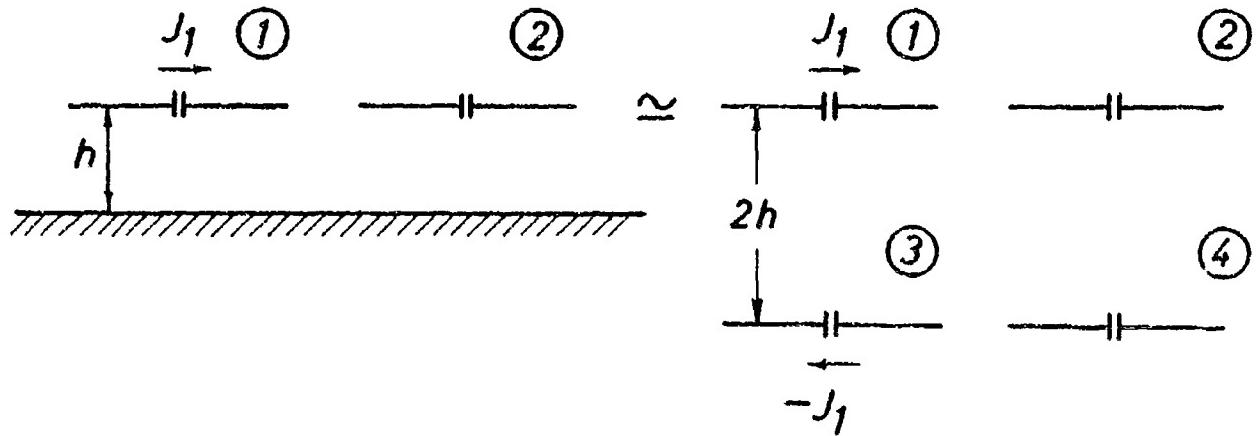


Fig. 13b. Mutual impedance between dipoles above plate.